

# Naturally Small Seesaw Neutrino Mass with No New Physics Beyond the TeV Scale

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## **Abstract**

If there is no new physics beyond the TeV energy scale, such as in a theory of large extra dimensions, the smallness of the seesaw neutrino mass, i.e.  $m_\nu = m_D^2/m_N$ , cannot be explained by a very large  $m_N$ . In contrast to previous attempts to find an alternative mechanism for a small  $m_\nu$ , I show how a solution may be obtained in a simple extension of the Standard Model, without using any ingredient supplied by the large extra dimensions. It is also experimentally testable at future accelerators.

In the minimal Standard Model of particle interactions, neutrinos are massless but they may acquire naturally small Majorana masses through the effective dimension-five operator [1]

$$\frac{1}{\Lambda}(\nu_i\phi^0 - l_i\phi^+)(\nu_j\phi^0 - l_j\phi^+), \quad (1)$$

where  $\Lambda$  is an effective large mass scale, and  $\Phi = (\phi^+, \phi^0)$  is the usual Higgs doublet with a nonzero vacuum expectation value,  $\langle\phi^0\rangle = v$ . The most common realization of this operator is the canonical seesaw mechanism [2], where three heavy (right-handed) singlet neutrinos  $N_i$  are introduced so that

$$m_\nu = \frac{m_D^2}{m_N}, \quad (2)$$

with  $m_D = fv$ , hence  $\Lambda = m_N/f^2$  in Eq. (1). Given that  $m_\nu$  is at most of order 1 eV and  $f$  should not be too small, the usual thinking is that  $m_N$  has to be very large, i.e.  $m_N \gg v$ . As such, this famous mechanism must be accepted on faith, because there cannot be any direct experimental test of its validity.

Consider now the possibility that there is no new physics beyond the TeV energy scale. This is an intriguing idea proposed recently in theories of large extra dimensions [3]. Since a large  $m_N$  is not available, the smallness of  $m_\nu$  in such theories is usually accomplished [4, 5] by putting  $N$  in the bulk and then pairing it with  $\nu$  to form a Dirac neutrino so that its mass is suppressed by the volume of the extra dimensions. Another approach is to break lepton number spontaneously in the bulk through a scalar singlet [6] which “shines” in our world as a small vacuum expectation value. This mechanism may then be combined with the triplet Higgs model of Majorana neutrino mass [7] to allow direct experimental determination of the relative magnitude of each element of the neutrino mass matrix from  $\xi^{++} \rightarrow l_i^+ l_j^+$  decay [8].

Instead of using an ingredient supplied by the large extra dimensions, I show in the following how Eq. (2) may be realized naturally with  $m_N$  of order 1 TeV in a simple extension

of the Standard Model. This means that  $m_D$  should be small, i.e.  $m_D \ll 10^2$  GeV. If it comes from  $\phi^0$  as in the Standard Model, that would not be natural; but as shown below, it will come instead from another doublet with a naturally small vacuum expectation value [9]. This new realization of the seesaw mechanism will allow direct experimental tests of its validity, as discussed below.

Consider the minimal Standard Model with three lepton families:

$$\begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L \sim (1, 2, -1/2), \quad l_{iR} \sim (1, 1, -1), \quad (3)$$

where their transformations under the standard  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge group are denoted as well. I now add three neutral fermion singlets

$$N_{iR} \sim (1, 1, 0), \quad (4)$$

but instead of assigning them lepton number  $L = 1$ , so they can pair up with the lepton doublet through the interaction  $\bar{N}_R(\nu_L\phi^0 - l_L\phi^+)$ , I assign them  $L = 0$  to forbid this Yukawa term. To complete my model, a new scalar doublet

$$\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \sim (1, 2, 1/2) \quad (5)$$

is introduced with lepton number  $L = -1$ . Hence the terms

$$\frac{1}{2}M_i N_{iR}^2 + f_{ij}\bar{N}_{iR}(\nu_{jL}\eta^0 - l_{jL}\eta^+) + h.c. \quad (6)$$

appear in the Lagrangian. The effective operator of Eq. (1) for neutrino mass is then replaced by one with  $\eta$  instead of  $\phi$ , and if  $\langle\eta^0\rangle = u$  is naturally small, the corresponding scale  $\Lambda$  will not have to be so large and  $M_i$  of Eq. (6) may indeed be of order 1 TeV.

The Higgs potential of this model is given by

$$\begin{aligned} V = & m_1^2\Phi^\dagger\Phi + m_2^2\eta^\dagger\eta + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\eta^\dagger\eta)^2 \\ & + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \mu_{12}^2(\Phi^\dagger\eta + \eta^\dagger\Phi), \end{aligned} \quad (7)$$

where the  $\mu_{12}^2$  term breaks  $L$  explicitly but softly [10]. Note that given the particle content of this model, the  $\mu_{12}^2$  term is the only possible soft term which also breaks  $L$ .

For  $\langle\phi^0\rangle = v$  and  $\langle\eta^0\rangle = u$ , the equations of constraint are

$$v[m_1^2 + \lambda_1 v^2 + (\lambda_3 + \lambda_4)u^2] + \mu_{12}^2 u = 0, \quad (8)$$

$$u[m_2^2 + \lambda_2 u^2 + (\lambda_3 + \lambda_4)v^2] + \mu_{12}^2 v = 0. \quad (9)$$

Consider the case

$$m_1^2 < 0, \quad m_2^2 > 0, \quad |\mu_{12}^2| \ll m_2^2, \quad (10)$$

then

$$v^2 \simeq \frac{-m_1^2}{\lambda_1}, \quad u \simeq \frac{-\mu_{12}^2 v}{m_2^2 + (\lambda_3 + \lambda_4)v^2}. \quad (11)$$

Hence  $u$  may be very small compared to  $v$  ( $= 174$  GeV). For example, if  $m_2 \sim 1$  TeV,  $|\mu_{12}^2| \sim 10$  GeV<sup>2</sup>, then  $u \sim 1$  MeV. The relative smallness of  $|\mu_{12}^2|$  may be attributed to the fact that it corresponds to the explicit breaking of lepton number in  $V$  of Eq. (7). [The usual argument here is that if  $|\mu_{12}^2|$  were zero, then the model's symmetry is increased, i.e. lepton number would be unbroken. Hence the assumption that it is small compared to  $|m_1^2|$  or  $m_2^2$  is “natural”. If  $|\mu_{12}^2|$  were much larger, then  $u$  would be proportionally larger, and since  $m_\nu$  scales as  $u^2$ , neutrino masses would be too large. It would also mean that the two scalar doublets mix to a substantial degree, which is not the case here, as discussed later in the paper. If  $|\mu_{12}^2|$  were much smaller, then neutrino masses would be too small to account for the present observation of neutrino oscillations.]

The  $6 \times 6$  mass matrix spanning  $[\nu_e, \nu_\mu, \nu_\tau, N_1, N_2, N_3]$  is now given by

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & 0 & 0 & f_{e1}u & f_{e2}u & f_{e3}u \\ 0 & 0 & 0 & f_{\mu1}u & f_{\mu2}u & f_{\mu3}u \\ 0 & 0 & 0 & f_{\tau1}u & f_{\tau2}u & f_{\tau3}u \\ f_{e1}u & f_{\mu1}u & f_{\tau1}u & M_1 & 0 & 0 \\ f_{e2}u & f_{\mu2}u & f_{\tau2}u & 0 & M_2 & 0 \\ f_{e3}u & f_{\mu3}u & f_{\tau3}u & 0 & 0 & M_3 \end{bmatrix}. \quad (12)$$

The mixing between  $\nu$  and  $N$  is thus of order  $fu/M$ , which will allow the physical  $N$  to decay through its small component of  $\nu$  to  $l^\pm W^\mp$ . The effective mass matrix spanning the three light neutrinos is then

$$\mathcal{M}_{ij} = \sum_k \frac{f_{ik} f_{jk} u^2}{M_k} \quad (13)$$

and is of order 1 eV if  $f$  is of order unity.

There are five physical Higgs bosons:

$$h^\pm = \frac{v\eta^\pm - u\phi^\pm}{\sqrt{v^2 + u^2}}, \quad A = \frac{\sqrt{2}(v\text{Im}\eta^0 - u\text{Im}\phi^0)}{\sqrt{v^2 + u^2}}, \quad (14)$$

$$h_1^0 \simeq \frac{\sqrt{2}(v\text{Re}\phi^0 + u\text{Re}\eta^0)}{\sqrt{v^2 + u^2}}, \quad h_2^0 \simeq \frac{\sqrt{2}(v\text{Re}\eta^0 - u\text{Re}\phi^0)}{\sqrt{v^2 + u^2}}, \quad (15)$$

with masses given by

$$m_{h^\pm}^2 = m_2^2 + \lambda_3 v^2 + (\lambda_2 - \lambda_4)u^2 - \mu_{12}^2 u/v, \quad (16)$$

$$m_A^2 = m_2^2 + (\lambda_3 + \lambda_4)v^2 + \lambda_2 u^2 - \mu_{12}^2 u/v, \quad (17)$$

$$m_{h_1^0}^2 = 2\lambda_1 v^2 + \mathcal{O}(u^2), \quad (18)$$

$$m_{h_2^0}^2 = m_2^2 + (\lambda_3 + \lambda_4)v^2 + \mathcal{O}(u^2). \quad (19)$$

From Eq. (15), it is clear that  $h_1^0$  behaves very much like the standard Higgs boson, as far as its coupling to all other particles are concerned. The new scalar particles of this model, i.e.  $h^\pm$ ,  $A$ , and  $h_2^0$  (all with mass  $\sim m_2$ ), as well as  $N_{iR}$  are now also accessible to direct experimental discovery in future accelerators. The key is of course Eq. (6).

Consider first the case  $m_2 > M_i$ . The decay chain

$$h^+ \rightarrow l_i^+ N_j, \quad \text{then } N_j \rightarrow l_k^\pm W^\mp, \quad (20)$$

will determine the relative magnitude of each element of  $\mathcal{M}_\nu$  in Eq. (12). Note that  $h^+ \rightarrow l_i^+ l_k^+ W^-$  can be a very distinct experimental signature. This direct test of the seesaw

mechanism as the source of neutrino mass will remove all uncertainties regarding the indirect determination of  $\mathcal{M}_\nu$  from neutrino-oscillation experiments.

Whereas  $h^\pm$  is readily produced through its electromagnetic interaction,  $h_2^0$  and  $A$  are only produced through their weak interactions, i.e.  $Z \rightarrow h_2^0 A$  and  $W^\pm \rightarrow h^\pm(h_2^0, A)$ . Their decay chain

$$h_2^0, A \rightarrow \nu N, \quad \text{then} \quad N \rightarrow l^\pm W^\mp, \quad (21)$$

is also less informative because the flavor of the neutrino in the first decay product cannot be identified experimentally.

Consider now the case  $M_i > m_2$ . The decay

$$N_i \rightarrow l_j^\pm h^\mp \quad (22)$$

will determine  $|f_{ij}|$  in Eqs. (6) and (12). The subsequent decay of  $h^\pm$  occurs through its small component of  $\phi^\pm$ , so it is dominated by the final states  $t\bar{b}$  or  $\bar{t}b$  and should be easily identifiable. The production of  $N$  in a hadron collider is difficult, but with an  $e^+e^-$  or  $\mu^+\mu^-$  collider, it can be produced easily in pairs through  $h^\pm$  exchange. The decay of the two  $N$ 's will include final states of the type  $l_i^+ l_j^+ b\bar{b}t\bar{t}$  which are very distinctive. Note that whether  $m_2 > M_i$  or  $M_i > m_2$ ,  $e^+e^-$  or  $\mu^+\mu^-$  production of  $N$  is possible. In the former case,  $N$  decays into  $l^\pm W^\mp$ ,  $\nu Z$ , and  $\bar{\nu}Z$ , whereas in the latter case, it decays into  $l^\pm h^\mp$ ,  $\nu h_2^0$ ,  $\bar{\nu} h_2^0$ ,  $\nu A$ , and  $\bar{\nu} A$  (with  $h_2^0$  and  $A$  both decaying into  $t\bar{t}$ ). Either possibility will allow the experimenter to determine  $|f_{ij}|$  and  $M_i$ , thereby obtaining the neutrino mass matrix up to an overall scale factor.

Lepton flavor violation (LFV) is a generic feature of all models of neutrino mass. In this model, there is no LFV at tree level for charged leptons. However, it does occur in one loop through  $\eta$  and  $N$  exchange. The extra scalar doublet  $(\eta^+, \eta^0)$  also contributes to the oblique parameters in precision electroweak measurements [11]. These contributions are

easily calculated [12]. For example, with  $m_2^2 \gg M_Z^2$ ,

$$\Delta S = \frac{1}{24\pi} \frac{\lambda_4 v^2}{m_2^2}, \quad (23)$$

$$\Delta T = \frac{1}{96\pi} \frac{1}{s^2 c^2 M_Z^2} \frac{\lambda_4^2 v^2}{m_2^2}, \quad (24)$$

where  $s^2 = \sin^2 \theta_W$ ,  $c^2 = \cos^2 \theta_W$ . They are clearly negligible and will not change the excellent experimental fit of the minimal Standard Model.

In summary, a new seesaw model of neutrino mass is proposed, where a second scalar doublet  $(\eta^+, \eta^0)$  with lepton number  $L = -1$  is added to the minimal Standard Model together with three neutral right-handed fermion singlets  $N_i$  with lepton number  $L = 0$ . Thus  $N_i$  is allowed to have a Majorana mass  $M_i$  as well as the interaction  $f_{ij} \bar{N}_{iR} (\nu_{jL} \eta^0 - l_{jL} \eta^+)$ . Hence  $m_\nu$  is proportional to  $\langle \eta^0 \rangle^2 / M_i$  and if  $\langle \eta^0 \rangle \ll \langle \phi^0 \rangle$ ,  $M_i$  may be of order 1 TeV and be observable experimentally. This is accomplished with the Higgs potential of Eq. (7) where  $L$  is broken explicitly and uniquely with the soft term  $\Phi^\dagger \eta + \eta^\dagger \Phi$ .

The decay of  $N_i$  into a charged lepton together with a charged Higgs boson or  $W$  boson will determine the relative magnitude of each element of the neutrino mass matrix. Just as the discovery of the standard Higgs boson would settle the question of how quarks and leptons acquire mass, the discovery of  $N_i$  and the new scalar doublet of this model would settle the question of how neutrinos acquire mass, and remove all uncertainties regarding the indirect determination of  $\mathcal{M}_\nu$  from neutrino-oscillation experiments.

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